Ch8. Rotational Kinematics
Rotational Motion and Angular Displacement

**Angular displacement**: When a rigid body rotates about a fixed axis, the angular displacement is the angle $\Delta \theta$ swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly. By convention, the angular displacement is positive if it is counterclockwise and negative if it is clockwise.

**SI Unit of Angular Displacement**: radian (rad)

Angular displacement is expressed in one of three units:

1. **Degree** (1 full turn $360^\circ$ degree)
2. **Revolution (rev)** $\rightarrow$ RPM
3. **Radian (rad)** SI unit
Example 1. A bicycle wheel has rotated 4.5 revolutions. How many radians has it rotated?
8-1 Angular Quantities

In purely rotational motion, all points on the object move in circles around the axis of rotation ("O"). The radius of the circle is \( r \). All points on a straight line drawn through the axis move through the same angle in the same time. The angle \( \theta \) in radians is defined:

\[
\theta = \frac{l}{r} \quad (8-1a)
\]

where \( l \) is the arc length.

Example 2. A particular bird's eye can just distinguish between objects that subtend an angle no smaller than about \( 3 \times 10^{-4} \) rad.

a. How many degrees is this?

b. What is the smallest object a bird can distinguish when flying at a height of 100 m?
Linear speed, which we simply called speed in previous chapters, is the distance traveled per unit of time. A point on the outside edge of a merry-go-round or turntable travels a greater distance in one complete rotation than a point nearer the center. Traveling a greater distance in the same time means a greater speed. Linear speed is greater on the outer edge of a rotating object than it is closer to the axis. The linear speed of something moving along a circular path can be called tangential speed because the direction of motion is tangent to the circumference of the circle. For circular motion, we can use the terms linear speed and tangential speed interchangeably. Units of linear or tangential speed are usually m/s or km/h.

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Rotational speed (sometimes called angular speed) involves the number of rotations or revolutions per unit of time. All parts of the rigid merry-go-round and turntable turn about the axis of rotation in the same amount of time. Thus, all parts share the same rate of rotation, or the same number of rotations or revolutions per unit of time. It is common to express rotational rates in revolutions per minute (RPM). For example, most phonograph turntables, which were common in mom and dad's time, rotate at $33\frac{1}{2}$ RPM. A ladybug sitting anywhere on the surface of the turntable revolves at $33\frac{1}{2}$ RPM.
DEFINITION OF AVERAGE ANGULAR VELOCITY

Average angular velocity

\[ \bar{\omega} = \frac{\text{Angular displacement}}{\text{Elapsed time}} \]

\[ \bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{\Delta \theta}{\Delta t} \]

*SI Unit of Angular Velocity: radian per second (rad/s)*
(a) When the turntable rotates, a point farther from the center travels a longer path in the same time and has a greater tangential speed. (b) A ladybug twice as far from the center moves twice as fast.
Tangential speed and rotational speed are related. Have you ever ridden on a big, round, rotating platform in an amusement park? The faster it turns, the faster your tangential speed. This makes sense; the greater the RPMs, the faster your speed in meters per second. We say that tangential speed is directly proportional to rotational speed at any fixed distance from the axis of rotation.
The tangential speed of each person is proportional to the rotational speed of the platform multiplied by the distance from the central axis.

Tangential speed, unlike rotational speed, depends on radial distance (the distance from the axis). At the very center of the rotating platform, you have no speed at all; you merely rotate. But, as you approach the edge of the platform, you find yourself moving faster and faster. Tangential speed is directly proportional to distance from the axis for any given rotational speed.
So we see that tangential speed is proportional to both radial distance and rotational speed.²

Tangential speed $\sim$ radial distance $\times$ rotational speed

In symbol form,

$$v \sim r\omega$$

where $v$ is tangential speed and $\omega$ (Greek letter omega) is rotational speed. You move faster if the rate of rotation increases (bigger $\omega$). You also move faster if you move farther from the axis (bigger $r$). Move out twice as far from the rotational axis at the center and you move twice as fast. Move out 3 times as far and you have 3 times as much tangential speed. If you find yourself in any kind of rotating system, your tangential speed depends on how far you are from the axis of rotation.

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**Example 3.** A gymnast on a high bar swings through two revolutions in a time of 1.90s.

a. Find the average **angular velocity** (in rad/s) of the gymnast.

b. If the gymnast is 1.75 m tall, what is the linear velocity of their feet?
In linear motion, a changing velocity means that an acceleration is occurring. Such is also the case in rotational motion; a changing angular velocity means that an angular acceleration is occurring.

**CONCEPTS AT A GLANCE** The idea of angular acceleration describes how rapidly or slowly the angular velocity changes during a given time interval.
DEFINITION OF AVERAGE ANGULAR ACCELERATION

\[
\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}
\]

\[
\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta \omega}{\Delta t}
\]

**SI Unit of Average Angular Acceleration:** radian per second squared (rad/s²)

A jet awaiting clearance for takeoff is momentarily stopped on the runway. As seen from the front of one engine, the fan blades are rotating with an angular velocity of $-110$ rad/s, where the negative sign indicates a clockwise rotation.

As the plane takes off, the angular velocity of the blades reaches $-330$ rad/s in a time of 14 s. Find the average angular velocity, assuming that the orientation of the rotating object is given by.....
8-1 Angular Quantities

If the angular velocity of a rotating object changes, it has a tangential acceleration:

$$a_{\text{tan}} = r\alpha$$  \hspace{1cm} (8-5)

Even if the angular velocity is constant, each point on the object has a centripetal acceleration:

$$a_{\text{R}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$  \hspace{1cm} (8-6)

8-1 Angular Quantities

Here is the correspondence between linear and rotational quantities:

<table>
<thead>
<tr>
<th>Linear</th>
<th>Type</th>
<th>Rotational</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>displacement</td>
<td>$\theta$</td>
<td>$x = r\theta$</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
<td>$\omega$</td>
<td>$v = r\omega$</td>
</tr>
<tr>
<td>$a_{\text{tan}}$</td>
<td>acceleration</td>
<td>$\alpha$</td>
<td>$a_{\text{tan}} = r\alpha$</td>
</tr>
</tbody>
</table>

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8-1 Angular Quantities

The frequency is the number of complete revolutions per second:

\[ f = \frac{\omega}{2\pi} \]

Frequencies are measured in hertz.

\[ 1 \text{ Hz} = 1 \text{ s}^{-1} \]

The period is the time one revolution takes:

\[ T = \frac{1}{f} \quad \text{(8-8)} \]

Example 4. A carousel is initially at rest. It is given a constant acceleration of \( a = 0.060 \text{ rad/s}^2 \), which increases its angular velocity for 8.0 seconds. The distance of a child from the axis of rotation is 2.5 m. Determine

a. The angular velocity of the carousel.

b. The linear velocity of the child.

c. The tangential acceleration of the child.
Attachments

08_RotationalSpeed_VID.mov