Bellringer:

1. In quadrilateral $PQRS$ below, sides $PS$ and $QR$ are parallel for what value of $x$?

   ![Diagram]

   A. 158
   B. 132
   C. 120
   D. 110
   E. 70

2. When $x = 3$ and $y = 5$, by how much does the value of $3x^2 - 2y$ exceed the value of $2x^2 - 3y$?

   \[
   \begin{align*}
   3(3)^2 - 2(5) &= 3(9) - 10 = 17 \\
   2(3)^2 - 3(5) &= 2(9) - 15 = 3
   \end{align*}
   \]
Questions to think about?

1. We exclude from a function's domain real numbers that cause division by __________.
   - Zero

2. Real numbers that result in a square root of ________________ are excluded from a function's domain.
   - negative numbers

Finding a Function’s Domain

If a function $f$ does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of $f(x)$ is a real number. Exclude from a function’s domain real numbers that cause division by zero and real numbers that result in an even root, such as a square root, of a negative number.
Examples to think about and how to write the domain of the functions.

\[ f(x) = \frac{1}{x - 3}. \]

What do we have to think about?

\[ x - 3 \neq 0 \]
\[ x \neq 3 \]
\[ (-\infty, 3) \cup (3, \infty) \]
Now consider a function involving a square root:

\[ g(x) = \sqrt{x - 3}. \]

\[
\begin{align*}
  x - 3 & \geq 0 \\
  x & \geq 3 \\
  [3, \infty)
\end{align*}
\]
You Try It!

**Check Point 1** Find the domain of each function:

a. \( f(x) = x^2 + 3x - 17 \)

b. \( g(x) = \frac{5x}{x^2 - 49} \)

c. \( h(x) = \sqrt{9x - 27} \)

d. \( j(x) = \frac{5x}{\sqrt{24 - 3x}} \)

a) \( \text{domain: } (-\infty, \infty) \)

b) \( x^2 - 49 \neq 0 \)

\( x^2 \neq 49 \)

\( x \neq \pm 7 \)

\( (-\infty, -7) \cup (-7, 7) \cup (7, \infty) \)

c) \( 9x - 27 \geq 0 \)

\( 9x \geq 27 \)

\( x \geq 3 \)

\( [3, \infty) \)

d) \( 24 - 3x > 0 \)

\(-3x > -24 \)

\( x < 8 \)

\( (-\infty, 8) \)
Combine functions using the algebra of functions, specifying domains.

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let \( f \) and \( g \) be two functions. The sum \( f + g \), the difference \( f - g \), the product \( fg \), and the quotient \( \frac{f}{g} \) are functions whose domains are the set of all real numbers common to the domains of \( f \) and \( g (D_f \cap D_g) \), defined as follows:

1. Sum: \[(f + g)(x) = f(x) + g(x)\]
2. Difference: \[(f - g)(x) = f(x) - g(x)\]
3. Product: \[(fg)(x) = f(x) \cdot g(x)\]
4. Quotient: \[\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0.\]
EXAMPLE

Combining Functions

Let \( f(x) = 2x - 1 \) and \( g(x) = x^2 + x - 2 \). Find each of the following functions:

- \( (f + g)(x) \)
- \( (f - g)(x) \)
- \( (fg)(x) \)
- \( \left( \frac{f}{g} \right)(x) \)

Determine the domain for each function.

a) \( (f + g)(x) = 2x - 1 + x^2 + x - 2 \)
   \[ = x^2 + 3x - 3 \quad \text{D: } (-\infty, \infty) \]

b) \( (f - g)(x) = 2x - 1 - (x^2 + x - 2) \)
   \[ = 2x - 1 - x^2 - x + 2 \]
   \[ = -x^2 + x + 1 \quad \text{D: } (-\infty, \infty) \]

c) \( (f \cdot g)(x) = (2x - 1)(x^2 + x - 2) \)
   \[ = 2x^3 + 2x^2 - 4x - x^2 - x + 2 \]
   \[ = 2x^3 + x^2 - 5x + 2 \quad \text{D: } (-\infty, \infty) \]

d) \( \left( \frac{f}{g} \right)(x) = \frac{2x - 1}{x^2 + x - 2} \)
   \[ = \frac{2x - 1}{(x + 2)(x - 1)} \]
   \[ (x + 2)(x - 1) \neq 0 \quad \text{D: } (-\infty, -2) \cup (-2, 1) \cup (1, \infty) \]

GREAT QUESTION!

Should I simplify a quotient function before finding its domain?

No. If the function \( \frac{f}{g} \) can be simplified, determine the domain before simplifying. All values of \( x \) for which \( g(x) = 0 \) must be excluded from the domain.