2013 Question 2

A 20 kg box on a horizontal frictionless surface is moving to the right at a speed of 4.0 m/s. The box hits and remains attached to one end of a spring of negligible mass whose other end is attached to a wall. As a result, the spring compresses a maximum distance of 0.50 m, and the box then oscillates back and forth.

(a)

i. The spring does work on the box from the moment the box first hits the spring to the moment the spring first reaches its maximum compression. Indicate whether the work done by the spring is positive, negative, or zero.

Positive  \checkmark  Negative  \_  Zero

Justify your answer.

The box will lose kinetic energy. Work is the change in kinetic energy. Since energy is lost, work is negative.

ii. Calculate the magnitude of the work described in part i.

\[ W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 \]

\[ W = 0 - \frac{1}{2} (20) (4)^2 \]

\[ W = -160 \text{ J} \]

(b) Calculate the spring constant of the spring.

Conservation of energy. The kinetic energy of the box is converted to potential energy of the spring.

\[ K_{\text{max}} = \frac{1}{2} k x^2 \]

\[ \frac{1}{2} (20) (4)^2 = \frac{1}{2} k (0.5)^2 \]

\[ k = \frac{1280}{(0.5)^2} \]

\[ k = 1280 \text{ N/m} \]

(c) Calculate the magnitude of the maximum acceleration of the box.

\[ F_s = \Sigma F \]

\[ K x = ma \]

\[ 1280 (0.5) = 20 a \]

\[ a = 32 \text{ m/s}^2 \]

(d) Calculate the frequency of the oscillation of the box.

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

\[ T = 2\pi \sqrt{\frac{20}{1280}} \]

\[ T = \frac{\pi}{0.785} \text{ s} \]

\[ f = \frac{1}{T} \]

\[ f = \frac{1}{\frac{1}{0.785}} \]

\[ f = 0.94 \text{ Hz} \]

\[ f = 1.27 \text{ Hz} \]
(c) Let \( x = 0 \) be the point where the box makes contact with the spring, with positive \( x \) directed toward the right.

i. On the axes below, sketch the kinetic energy, \( K \), of the oscillating box as a function of position \( x \) for the range \( x = -0.50 \text{ m} \) to \( x = +0.50 \text{ m} \).

When the box is in contact with the spring initially, that is maximum \( K \) is 160 J. When it goes to \( x = 0.5 \), \( K = 0 \). It gains \( K \) as it slides to the left back to \( x = 0 \). Then, as it slides back to the left, the max displacement is \( +0.5 \text{ m} \), where \( K = 0 \) again. Repeat a bunch of times. Remember that the \( K \) lost is equal to \( K \) gained, and \( U \) is gained by \( \frac{1}{2} k x^2 \). So, the loss of \( K \) is not linear, it is quadratic respect to \( x \). Or, \( K = 160 - \frac{1}{2} (1280) x^2 \).

So, the graph is a parabola, opening down.

ii. On the axes below, sketch the acceleration, \( a \), of the oscillating box as a function of position \( x \) for the range \( x = -0.50 \text{ m} \) to \( x = +0.50 \text{ m} \).

We know \( a = 0 \) when \( x = 0 \)

b/c the spring has not been displaced. When \( x = 0.5 \text{ m} \),
\[
a = \frac{-32}{1280} = \frac{-2}{1280} \text{ m/s}^2.
\]

When \( x = -0.5 \text{ m} \),
\[
a = \frac{32}{1280} = \frac{1}{1280} \text{ m/s}^2.
\]

Now, the slope.

We calculate \( a \) from \( \Sigma F = ma \) where \( F = kx \).

The force increases linearly with \( x \). So, \( a \) increases linearly with \( x \). Or, \( ma = -kx \) and \( a = -\frac{k}{m} x \). \( k \) and \( m \) are constants so, \( a = -\frac{1280}{20} x \), which is a linear function with a slope of \( -64 \).
Problem 2.

In the system of two blocks and a spring shown above, block 1 and block 2 are connected by a light string passing over a frictionless pulley of negligible mass. Block 1 rests on a frictionless surface. Block 1 has a mass of 3.0 kg, block 2 has a mass of 1.0 kg, and the spring constant is 20 N/m.

a. What is the stretch of the spring?

Since the block is in equilibrium, \( T = F_s \) and \( T = m_2g \), so

\[ F_s = mg \quad \text{or} \quad Kx = mg. \]

\[ \Rightarrow 20x = 10 \times 10 \]

\[ x = 0.5 \text{ m} \]

b. If the string breaks and block 1 is set into harmonic motion, what will its maximum speed be?

Law of conservation of energy. The stored \( U_s \) will convert to \( K. \)

\[ \frac{1}{2} Kx^2 = \frac{1}{2} m_1 v^2 \]

\[ 20(0.5)^2 = 3 v^2 \]

\[ v = 1.29 \text{ m/s} \]

c. If the string breaks and block 1 is set into harmonic motion, what will its period be?

\[ T = 2\pi \sqrt{\frac{m_1}{K}} \]

\[ T = 2\pi \sqrt{\frac{3}{20}} \]

\[ T = 2.435 \]

d. If the spring breaks, what will the maximum acceleration of the block system be?

\[ \Sigma F = ma \]

\[ m_2 g - T = m_2 a \]

\[ T = m_1 a \]

\[ m_2 g = m_1 a + m_2 a \]

\[ m_2 g = (m_1 + m_2) a \]

\[ a = \frac{m_2 g}{m_1 + m_2} = \frac{10}{1+3} = \frac{2.5 \text{ m/s}^2}{5} \]
2005B Question 2

A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to a point Q, which is 0.08 m above its lowest position, and released so that it oscillates with a small amplitude $\Theta$ between points P and Q as shown below.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

ii. When it is in motion at its lowest position

(b) Calculate the speed $v$ of the bob at its lowest position.

Conservation of Energy,

$$U_p + K_p = U_{bottom} + K_{bottom}$$

$$mg h_p = \frac{1}{2} m v_{bottom}^2$$

$$U_{bottom} = \sqrt{2gh_p}$$

$$v_{bottom} = \sqrt{2gh_p} = \sqrt{2(9.8)(0.08)}$$

$$v = 1.26 \text{ m/s}$$
(c) Calculate the tension in the string when the bob is passing through its lowest position.

Circular Motion. Centripetal force = \( ma \)

\[
\sum F = ma
\]

\[
T - Mg = \frac{mv^2}{r}
\]

\[
T = Mg + \frac{mv^2}{r}
\]

\[
T = 0.085 \times 10 + \frac{0.085 \times 1.26^2}{1.5}
\]

\[
T = 0.94 \text{ N}
\]

(d) Describe one modification that could be made to double the period of oscillation.

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

So, to make \( T \) increase, you would have to increase the length or take it to a place with less gravity. Both terms are under the square root, so you would need:

- The new length to be \( 4L \) so that \( \sqrt{\frac{4L}{g}} = 2 \sqrt{\frac{L}{g}} \). This would double the period.
- The new gravitational field to be \( \frac{1}{4}g \) so that \( \sqrt{\frac{L}{\frac{1}{4}g}} = 2 \sqrt{\frac{L}{g}} \). This would double the period.