Warm Up

1. Write the angles in order from smallest to largest.

\[ \angle X, \angle Z, \angle Y \]

2. The lengths of two sides of a triangle are 12 cm and 9 cm. Find the range of possible lengths for the third side. \[ 3 \text{ cm} < s < 21 \text{ cm} \]
Inequalities in Two Triangles

Objective

Apply inequalities in two triangles.
### Inequalities in Two Triangles

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<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
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<tr>
<td><strong>5-6-1</strong> Hinge Theorem</td>
<td>If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is across from the larger included angle.</td>
<td>$BC &gt; EF$</td>
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<td></td>
<td>$m\angle A &gt; m\angle D$</td>
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<tr>
<td><strong>5-6-2</strong> Converse of the Hinge Theorem</td>
<td>If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is across from the longer third side.</td>
<td>$m\angle J &gt; m\angle M$</td>
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<td>$GH &gt; KL$</td>
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</table>
Example 1A: Using the Hinge Theorem and Its Converse

Compare \( \angle BAC \) and \( \angle DAC \).

Compare the side lengths in \( \Delta ABC \) and \( \Delta ADC \).

\[
AB = AD \quad AC = AC \quad BC > DC
\]

By the Converse of the Hinge Theorem, \( m\angle BAC > m\angle DAC \).
Compare *EF* and *FG*.

Compare the sides and angles in Δ*EFH* angles in Δ*GFH*.

\[ \text{m} \angle GHF = 180^\circ - 82^\circ = 98^\circ \]

*EH = GH*  \quad *FH = FH*  \quad \text{m} \angle EHF > \text{m} \angle GHF

By the Hinge Theorem, *EF < GF*. 
Compare $m\angle EGH$ and $m\angle EGF$.

Compare the side lengths in $\triangle EGH$ and $\triangle EGF$.

$FG = HG \quad EG = EG \quad EF > EH$

By the Converse of the Hinge Theorem, $m\angle EGH < m\angle EGF$. 
Compare $BC$ and $AB$.

Compare the side lengths in $\triangle ABD$ and $\triangle CBD$.

$AD = DC \quad BD = BD \quad m\angle ADB > m\angle BDC$.

By the Hinge Theorem, $BC > AB$. 
1. Compare $m\angle ABC$ and $m\angle DEF$.

$\triangle ABC$: 
- $AB = 12$
- $BC = 6$
- $CA = 7$

$\triangle DEF$: 
- $DE = 10$
- $EF = 6$
- $DF = 7$

$m\angle ABC > m\angle DEF$

2. Compare $PS$ and $QR$.

$\triangle PQR$: 
- $PQ = 8$
- $QR = 8$
- $PR = 8$
- $\angle PQR = 100^\circ$
- $\angle QRP = 80^\circ$

$PS < QR$
3. Find the range of values for $z$.

$-3 < z < 7$
4. Write a two-column proof.

**Prove:** \( m\angle XYW < m\angle ZYW \)

**Given:** \( \overline{XY} \cong \overline{WZ}, XW < YZ \)

**Proof:**

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<th>Statements</th>
<th>Reasons</th>
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<tr>
<td>1. ( \overline{XY} \cong \overline{WZ}, XW &lt; YZ )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{YW} \equiv \overline{YW} )</td>
<td>2. Reflex. Prop. of ( \cong )</td>
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<td>3. ( m\angle XYW &lt; m\angle ZYW )</td>
<td>3. Conv. of Hinge Thm.</td>
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