Warm Up

1. Write a conditional from the sentence “An isosceles triangle has two congruent sides.”
   If a $\Delta$ is isosc., then it has 2 $\cong$ sides.

2. Write the contrapositive of the conditional “If it is Tuesday, then John has a piano lesson.”
   If John does not have a piano lesson, then it is not Tuesday.

3. Show that the conjecture “If $x > 6$, then $2x > 14$” is false by finding a counterexample.
   $x = 7$
Objectives

Apply inequalities in one triangle.
The positions of the longest and shortest sides of a triangle are related to the positions of the largest and smallest angles.

**Theorems**  
**Angle-Side Relationships in Triangles**

<table>
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<th>THEOREM</th>
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| **5-5-1** | If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.  
(In \( \triangle \), larger \( \angle \) is opp. longer side.) | \( AB > BC \) | \( m\angle C > m\angle A \) |
| **5-5-2** | If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.  
(In \( \triangle \), longer side is opp. larger \( \angle \).) | \( XZ > XY \) | \( m\angle Z > m\angle Y \) |
Example 2A: Ordering Triangle Side Lengths and Angle Measures

Write the angles in order from smallest to largest.

The shortest side is \( \overline{GH} \), so the smallest angle is \( \angle F \).

The longest side is \( \overline{FH} \) so the largest angle is \( \angle G \).

The angles from smallest to largest are \( \angle F, \angle H \) and \( \angle G \).
Write the sides in order from shortest to longest.

\[ m\angle R = 180^\circ - (60^\circ + 72^\circ) = 48^\circ \]

The smallest angle is \( \angle R \), so the shortest side is \( \overline{PQ} \).

The largest angle is \( \angle Q \), so the longest side is \( \overline{PR} \).

The sides from shortest to longest are \( \overline{PQ}, \overline{QR}, \) and \( \overline{PR} \).
Write the angles in order from smallest to largest.

The shortest side is $\overline{AC}$, so the smallest angle is $\angle B$.

The longest side is $\overline{AB}$, so the largest angle is $\angle C$.

The angles from smallest to largest are $\angle B$, $\angle A$, and $\angle C$. 
Check It Out! Example 2b

Write the sides in order from shortest to longest.

\[ m\angle E = 180^\circ - (90^\circ + 22^\circ) = 68^\circ \]

The smallest angle is \( \angle D \), so the shortest side is \( \overline{EF} \).

The largest angle is \( \angle F \), so the longest side is \( \overline{DE} \).

The sides from shortest to longest are \( \overline{EF}, \overline{DF}, \) and \( \overline{DE} \).
A triangle is formed by three segments, but not every set of three segments can form a triangle.

Segments with lengths of 7, 4, and 4 can form a triangle.

Segments with lengths of 7, 3, and 3 cannot form a triangle.
A certain relationship must exist among the lengths of three segments in order for them to form a triangle.

**Theorem 5-5-3  Triangle Inequality Theorem**

The sum of any two side lengths of a triangle is greater than the third side length.

\[
AB + BC > AC \\
BC + AC > AB \\
AC + AB > BC
\]
Example 3A: Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

7, 10, 19

\[ 7 + 10 \nleq 19 \]
\[ 17 \nleq 19 \]

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.
Example 3B: Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

2.3, 3.1, 4.6

\[ 2.3 + 3.1 \geq 4.6 \quad 2.3 + 4.6 \geq 3.1 \quad 3.1 + 4.6 \geq 2.3 \]

\[ 5.4 > 4.6 \quad 6.9 > 3.1 \quad 7.7 > 2.3 \]

Yes—the sum of each pair of lengths is greater than the third length.
Example 3C: Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

\( n + 6, \ n^2 - 1, \ 3n, \) when \( n = 4. \)

**Step 1** Evaluate each expression when \( n = 4. \)

\[
\begin{align*}
n + 6 & \quad n^2 - 1 & \quad 3n \\
4 + 6 & \quad (4)^2 - 1 & \quad 3(4) \\
10 & \quad 15 & \quad 12
\end{align*}
\]
Example 3C Continued

**Step 2** Compare the lengths.

\[
10 + 15 \gtrsim 12 \quad 10 + 12 \gtrsim 15 \quad 15 + 12 \gtrsim 10
\]

\[
25 > 12 \checkmark \quad 22 > 15 \checkmark \quad 27 > 10 \checkmark
\]

Yes—the sum of each pair of lengths is greater than the third length.
Tell whether a triangle can have sides with the given lengths. Explain.

8, 13, 21

\[ 8 + 13 \geq 21 \]
\[ 21 \neq 21 \]

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.
Tell whether a triangle can have sides with the given lengths. Explain.

6.2, 7, 9

\[6.2 + 7 > 9\]  \[13.2 > 9\]  ✔
\[6.2 + 9 > 7\]  \[15.2 > 7\]  ✔
\[7 + 9 > 6.2\]  \[16 > 6.2\]  ✔

Yes—the sum of each pair of lengths is greater than the third side.
Tell whether a triangle can have sides with the given lengths. Explain.

\( t - 2, 4t, t^2 + 1, \) when \( t = 4 \)

**Step 1** Evaluate each expression when \( t = 4 \).

\[
\begin{array}{ccc}
  t - 2 & 4t & t^2 + 1 \\
  4 - 2 & 4(4) & (4)^2 + 1 \\
  2 & 16 & 17 \\
\end{array}
\]
Step 2 Compare the lengths.

\[ 2 + 16 \geq 17 \quad 2 + 17 \geq 16 \quad 16 + 17 \geq 2 \]

\[ 18 > 17 \, \checkmark \quad 19 > 16 \, \checkmark \quad 33 > 2 \, \checkmark \]

Yes—the sum of each pair of lengths is greater than the third length.
Example 4: Finding Side Lengths

The lengths of two sides of a triangle are 8 inches and 13 inches. Find the range of possible lengths for the third side.

Let \( x \) represent the length of the third side. Then apply the Triangle Inequality Theorem.

\[
\begin{align*}
    x + 8 &> 13 \\
    x + 13 &> 8 \\
    8 + 13 &> x \\
    x &> 5 \\
    x &> -5 \\
    21 &> x
\end{align*}
\]

Combine the inequalities. So \( 5 < x < 21 \). The length of the third side is greater than 5 inches and less than 21 inches.
The lengths of two sides of a triangle are 22 inches and 17 inches. Find the range of possible lengths for the third side.

Let $x$ represent the length of the third side. Then apply the Triangle Inequality Theorem.

\[
x + 22 > 17 \quad x + 17 > 22 \quad 22 + 17 > x \\
x > -5 \quad x > 5 \quad 39 > x
\]

Combine the inequalities. So $5 < x < 39$. The length of the third side is greater than 5 inches and less than 39 inches.
Lesson Quiz: Part I

1. Write the angles in order from smallest to largest.
   \[\angle C, \angle B, \angle A\]

2. Write the sides in order from shortest to longest.
   \[\overline{DE}, \overline{EF}, \overline{DF}\]
3. The lengths of two sides of a triangle are 17 cm and 12 cm. Find the range of possible lengths for the third side.
   \[ 5 \text{ cm} < x < 29 \text{ cm} \]

4. Tell whether a triangle can have sides with lengths 2.7, 3.5, and 9.8. Explain.
   \[ \text{No; } 2.7 + 3.5 \text{ is not greater than } 9.8. \]