Warm Up
Write a conditional statement from each of the following.

1. The intersection of two lines is a point.
   If two lines intersect, then they intersect in a point.

2. An odd number is one more than a multiple of 2.
   If a number is odd, then it is one more than a multiple of 2.

3. Write the converse of the conditional “If Pedro lives in Chicago, then he lives in Illinois.” Find its truth value.
   If Pedro lives in Illinois, then he lives in Chicago; False.
Standard U2S5

Write and analyze biconditional statements.
When you combine a conditional statement and its converse, you create a *biconditional statement*.

A **biconditional statement** is a statement that can be written in the form \( p \) if and only if \( q \). This means “if \( p \), then \( q \)” and “if \( q \), then \( p \).”
The biconditional “p if and only if q” can also be written as “p iff q” or $p \iff q$.  

Writing Math

$p \iff q$ means $p \rightarrow q$ and $q \rightarrow p$. 
Example 1A: Identifying the Conditionals within a Biconditional Statement

Write the conditional statement and converse within the biconditional.

An angle is obtuse if and only if its measure is greater than $90^\circ$ and less than $180^\circ$.

Let $p$ and $q$ represent the following.

$p$: An angle is obtuse.
$q$: An angle’s measure is greater than $90^\circ$ and less than $180^\circ$. 
Example 1A Continued

Let \( p \) and \( q \) represent the following.

\( p: \text{An angle is obtuse.} \)

\( q: \text{An angle’s measure is greater than } 90^\circ \text{ and less than } 180^\circ. \)

The two parts of the biconditional \( p \iff q \) are \( p \rightarrow q \) and \( q \leftarrow p \).

Conditional: \textbf{If an } \angle \textbf{ is obtuse, then its measure is greater than } 90^\circ \text{ and less than } 180^\circ.

Converse: \textbf{If an angle's measure is greater than } 90^\circ \text{ and less than } 180^\circ, \text{ then it is obtuse.}
Example 1B: Identifying the Conditionals within a Biconditional Statement

Write the conditional statement and converse within the biconditional.

A solution is neutral $\iff$ its pH is 7.

Let $x$ and $y$ represent the following.

$x$: A solution is neutral.

$y$: A solution’s pH is 7.
Let $x$ and $y$ represent the following.

$x$: A solution is neutral.
$y$: A solution’s pH is 7.

The two parts of the biconditional $x \iff y$ are $x \rightarrow y$ and $y \rightarrow x$.

Conditional: If a solution is neutral, then its pH is 7.
Converse: If a solution’s pH is 7, then it is neutral.
Check It Out! Example 1b

Write the conditional statement and converse within the biconditional.

Cho is a member if and only if he has paid the $5 dues.

Let $x$ and $y$ represent the following.

$x$: Cho is a member.

$y$: Cho has paid his $5 dues.

The two parts of the biconditional $x \iff y$ are $x \rightarrow y$ and $y \rightarrow x$.

Conditional: If Cho is a member, then he has paid the $5 dues.
Converse: If Cho has paid the $5 dues, then he is a member.
Example 2: Identifying the Conditionals within a Biconditional Statement

For each conditional, write the converse and a biconditional statement.

A. If $5x - 8 = 37$, then $x = 9$.
   Converse: If $x = 9$, then $5x - 8 = 37$.
   Biconditional: $5x - 8 = 37$ if and only if $x = 9$.

B. If two angles have the same measure, then they are congruent.
   Converse: If two angles are congruent, then they have the same measure.
   Biconditional: Two angles have the same measure if and only if they are congruent.
Check It Out! Example 2a

For the conditional, write the converse and a biconditional statement.

If the date is July 4th, then it is Independence Day.

Converse: If it is Independence Day, then the date is July 4th.

Biconditional: It is July 4th if and only if it is Independence Day.
For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.
Example 3A: Analyzing the Truth Value of a Biconditional Statement

Determine if the biconditional is true. If false, give a counterexample.

A rectangle has side lengths of 12 cm and 25 cm if and only if its area is 300 cm².
Example 3A: Analyzing the Truth Value of a Biconditional Statement

Conditional: If a rectangle has side lengths of 12 cm and 25 cm, then its area is 300 cm$^2$. The conditional is true.

Converse: If a rectangle’s area is 300 cm$^2$, then it has side lengths of 12 cm and 25 cm. The converse is false.

If a rectangle’s area is 300 cm$^2$, it could have side lengths of 10 cm and 30 cm. Because the converse is false, the biconditional is false.
Example 3B: Analyzing the Truth Value of a Biconditional Statement

Determine if the biconditional is true. If false, give a counterexample.

A natural number $n$ is odd $\iff n^2$ is odd.

Conditional: If a natural number $n$ is odd, then $n^2$ is odd.
The conditional is true.

Converse: If the square $n^2$ of a natural number is odd, then $n$ is odd.
The converse is true.

Since the conditional and its converse are true, the biconditional is true.
In geometry, biconditional statements are used to write definitions.

A definition is a statement that describes a mathematical object and can be written as a true biconditional.
In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments.

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A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.
Helpful Hint

Think of definitions as being reversible. Postulates, however are not necessarily true when reversed.
Example 4: Writing Definitions as Biconditional Statements

Write each definition as a biconditional.

A. A pentagon is a five-sided polygon.
A figure is a pentagon if and only if it is a 5-sided polygon.

B. A right angle measures 90°.
An angle is a right angle if and only if it measures 90°.
Write each definition as a biconditional.

4a. A quadrilateral is a four-sided polygon.
   A figure is a quadrilateral if and only if it is a 4-sided polygon.

4b. The measure of a straight angle is $180^\circ$.
   An $\angle$ is a straight $\angle$ if and only if its measure is $180^\circ$. 